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ON THE COMPLEXITY OF THE DISJOINT PATHS PROBLEM

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In this paper we consider the disjoint paths problem. Given a graph G and a subset S of the edge-set of G the problem is to decide whether there exists a family $\mathcal{F}$ of disjoint circuits in G each containing exactly one edge of S such that every edge in S belongs to a circuit in $\mathcal{C}$. By a well-known theorem of P. Seymour the edge-disjoint paths problem is polynomially solvable for Eulerian planar graphs G. We show that (assuming $\mathbf{P} \neq \mathbf{NP}$) one can drop neither planarity nor the Eulerian condition on G without losing polynomial time solvability. We prove the **NP**-completeness of the planar edge-disjoint paths problem by showing the **NP**-completeness of the vertex disjoint paths problem for planar graphs with maximum vertex-degree three. This disproves (assuming $\mathbf{P} \neq \mathbf{NP}$) a conjecture of A. Schrijver concerning the existence of a polynomial time algorithm for the planar vertex-disjoint paths problem. Furthermore we present a counterexample to a conjecture of A. Frank. This conjecture would have implied a polynomial algorithm for the planar edge-disjoint paths problem. Moreover we derive a complete characterization of all minor-closed classes of graphs for which the disjoint paths problem is polynomially solvable. Finally we show the **NP**-completeness of the half-integral relaxation of the edge-disjoint paths problem. This implies an answer to the long-standing question whether the edge-disjoint paths problem is polynomially solvable for Eulerian graphs.

1. Introduction

It has been a long-standing unsolved problem whether there exists a polynomial algorithm for the integral planar multicommodity flow problem. Recently A. Sebő [8] gave an algorithm for this problem which is polynomial for a bounded number of demand edges. We show in this paper that the planar edge-disjoint paths problem (which is a special case of the integral planar multicommodity flow problem) is **NP**-complete. We do this by showing the **NP**-completeness of the planar vertex-disjoint paths problem for graphs G with maximum vertex-degree three. The planar vertex-disjoint paths problem was conjectured by A. Schrijver [6] to be polynomially solvable. Incidentally, we obtain a complete characterization of all minor-closed classes of graphs for which the disjoint paths problem is polynomial time solvable. We present a counterexample to a conjecture of A. Frank (mentioned in Sebő [7]) which would have implied a polynomial time algorithm for the integral planar multicommodity flow problem.

The integral multicommodity flow problem is known to be polynomial time solvable for planar Eulerian graphs. This is a well-known result of P. Seymour [9]. As mentioned above one cannot expect to get a polynomial algorithm for the planar case (dropping the Eulerian condition). We show that on the other hand neither one can expect to get a polynomial algorithm for the Eulerian case

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(dropping planarity). In fact we show the **NP**-completeness of the half-integral relaxation of the edge-disjoint paths problem. It has been a long-standing open question whether the half-integral relaxation of the edge-disjoint paths problem has a good characterization (which was conjectured by P. Seymour [9]).

2. Preliminaries

The central topics of this paper are the following disjoint paths problems:

Problem:	Edge-disjoint (resp. vertex-disjoint) paths problem.
Instance:	Graph $G=(V,S\cup D)$.
Question:	Do there exist edge-disjoint (resp. vertex-disjoint) circuits $C_1,\dots,C_{\|D\|}$ in G with $\|E(C_i)\cap D\|=1, 1\leq i\leq \|D\|$?

The edges of S and D are called "supply"- and "demand"- edges, respectively. The planar edge-/vertex-disjoint paths problem is the restriction of the general edge-/vertex-disjoint paths problem to planar graphs G.

There is a natural relaxation of the edge-disjoint paths problem called the multicommodity flow problem. Let $G=(V,S\cup D)$ be a graph with non-negative capacities $c(s)$ and requests $r(d)$ on supply-edges s and demand-edges d, respectively. One way to formulate the multicommodity flow problem is by a linear program:

Let A be the 0,1-matrix with rows corresponding to the supply-edges and columns corresponding to the circuits C of G containing exactly one demand edge. An entry (s,C) is 1 if and only if $s\in E(C)$. Similarly let B the 0,1-matrix with rows corresponding to the demand-edges $d\in D$ and columns corresponding to circuits as above. An entry (d,C) of B is 1 if and only if d is the demand edge in C. The multicommodity flow problem is to decide whether the linear program $Ax\leq c, Bx=r, x\geq 0$ has a solution.

The integral multicommodity flow problem and the half-integral relaxation of the edge-disjoint paths problem are derived from their respective linear programming formulations by imposing (half-)integrality constraints on the variables x.

3. The planar disjoint paths problem

Theorem 1. *The planar vertex-disjoint paths problem is* **NP**-*complete.*

Proof. We are going to reduce planar 3SAT to the problem in question. Recall that planar 3SAT is the restriction of 3SAT to instances $\mathscr{C}$ (considered as a set of clauses) with planar graph $G(\mathscr{C})$: the vertex-set of $G(\mathscr{C})$ is the union of $\mathscr{C}$ and the set V of variables occuring in $\mathscr{C}$ and a pair vC belongs to the edge-set of $G(\mathscr{C})$ if the variable v occurs in the clause C. Planar 3SAT is known to be **NP**-complete (cf. Lichtenstein [1]).

Planar 3SAT remains **NP**-complete even if restricted to instances where every variable $v\in V$ occurs in exactly three clauses. To see this take a planar embedding of $G(\mathscr{C})$, for some instance $\mathscr{C}$ of planar 3SAT. Let $vC_1,\dots,vC_k$, in clockwise order according to the planar embedding, be the edges adjacent to variable v in the graph

$G(\mathscr{C})$. Now introduce new variables $v_1, \ldots, v_k$ and clauses $\{v_k, \neg v_1\}$ and $\{v_i, \neg v_{i+1}\}$, $i = 1 \ldots k-1$, and replace literals $v, \neg v$ in clauses C_i by literals $v_i, \neg v_i$, respectively, for $i = 1, \ldots, k$. It is easily seen that the modified formula $\mathscr{C}'$ is satisfiable if and only if $\mathscr{C}$ is satisfiable. Again, $G(\mathscr{C}')$ is a planar graph. In case $k > 3$, $\mathscr{C}'$ has less variables with not exactly three occurences.

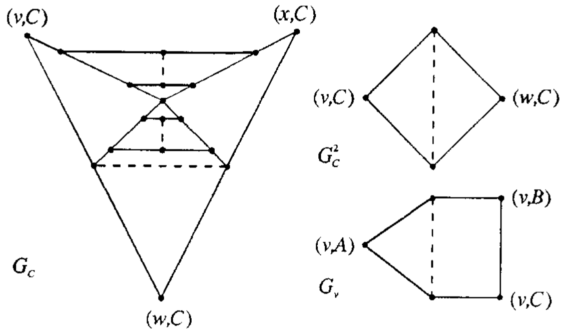

Fig. 1. The Gadgets G_C, G_C^2 and G_v

Let $\mathscr{C}$ be an instance of planar 3SAT with exactly three occurences for every variable. Whithout loss of generality we may assume that every variable of $\mathscr{C}$ occurs at least once negative and once positive in $\mathscr{C}$. Replacing the variables v with two negative occurences by their negations we obtain a formula which is satisfiable if and only if the original one is. In the new formula every of its variables occurs exactly once negated. We assume that this is the case for $\mathscr{C}$.

Now we built up an instance $G'(\mathscr{C})$ of the planar vertex-disjoint paths problem using the gadgets depicted in Figure 1 (dashed lines in figures refer to demand edges).

For every edge vC of the graph $G(\mathscr{C})$ we take a vertex (v, C). For every vertex of the graph $G(\mathscr{C})$ we take an appropriate gadget according to the following rules in such a way that vertices without name in Figure 1 do not belong to the set $VE := \{(v, C) \mid vC \in E(G(\mathscr{C}))\}$ and gadgets intersect only in vertices of VE.

For a variable v that occurs negatively in clause A and positively in the clauses B, C we take the gadget G_v as depicted in Figure 1.

For a clause C in which variables v, w, x occur, we take a gadget G_C as depicted in the same figure. For a clause C with only two literals in which variables v, w occur, we take a gadget G_C^2.

By the planarity of the graph $G(\mathscr{C})$ it is clear that the graph $G'(\mathscr{C})$ is planar. Obviously $G'(\mathscr{C})$ can be constructed in polynomial time.

Claim. *The given propositional formula $\mathscr{C}$ is satisfiable if and only if $G'(\mathscr{C})$ is a feasible instance of the vertex-disjoint paths problem.*

Let us first make some observations about the gadgets G_C that are easy to verify. The analogous statements about gadgets G_C^2 are trivial

i) The gadget G_C is a feasible instance of the planar vertex-disjoint paths problem.
ii) G_C remains feasible if any two (one) of the vertices $(v,C),(w,C),(x,C)$ are removed from G_C (notice that no demand-edge is incident to a vertex $(v,C) \in VE$).
iii) G_C turns unfeasible if all three vertices $(v,C),(w,C),(x,C)$ are removed from G_C.
iv) if any further demand-edge between two vertices of $V(G_C) \cap VE$ is added to G_C the gadget turns unfeasible.

Convention In the following G_C always means G_C or G_C^2.

Now assume that we have a truth assignment that satisfies the formula $\mathscr{C}$. Using the truth assignment we will prove that $G'(\mathscr{C})$ is feasible.

For the demand-edges in the gadget G_v (cf. Figure 1) we choose the circuit in G_v through the demand-edge and vertex (v,A) if the variable v is set true by the given truth assignment. We choose the circuit in G_v through the demand-edge and vertices (v,B) and (v,C) if v is set false. Notice that v occurs negatively in clause A and positively in the clauses B and C. In this manner we choose for every variable a circuit. Clearly all these circuits are vertex-disjoint.

Now we delete the vertices of the choosen circuits from the graph $G'(\mathscr{C})$. Since the given truth assignment satisfies $\mathscr{C}$ for every clause C at least one of the vertices of the gadget $V(G_C)$ does not belong to the deleted vertices. Thus by observation ii) the remaining part of G_C is feasible. Since the gadgets G_C and $G_{C'}$ are disjoint for distinct clauses C and C', it follows that $G'(\mathscr{C})$ is feasible.

Assume in G there exists a family $\mathscr{F}$ of $|D|$ vertex-disjoint circuits each of which contains exactly one demand-edge. The circuit in $\mathscr{F}$ containing the demand edge in G_v has to belong completely to the gadget G_v. Otherwise this circuit has to pass one of the gadgets G_C. By observation iv) and the fact that the gadgets G_C are linked to the rest of the graph only by three vertices this can't happen.

We define a truth assignment according to the circuits in $\mathscr{F}$ containing the demand-edges belonging to variable gadgets. A variable v is set true if the cicuit in $\mathscr{F}$ containing the demand edge in G_v containes the vertex (v,A) where A is the clause in which V occurs negated. Otherwise v is set false (notice that a cicuit in G_v that containes the demand-edge containes either (v,A) or (v,B)).

Let C be a clause of $\mathscr{C}$. We show that the truth assignment we have just defined satisfies C.

Let $\mathscr{F}_{\mathscr{C}}$ be the subfamily of $\mathscr{F}$ containing the circuits through demand-edges of G_C. An easy consequence of observation iii) is that at least one circuit of $\mathscr{F}_{\mathscr{C}}$ containes a vertex of (v,C) for a variable v that occurs in C. Thus for this variable v the circuit in $\mathscr{F}$ containing the demand-edge of G_v can not contain the vertex (v,C). Thus if v occurs negatively in C, v was set false whereas if v occurs positively in C, v was set true. Thus C and hence $\mathscr{C}$ is satisfied by our truth assignment. ∎

Theorem 2. *The planar vertex-disjoint paths problem restricted to graphs G with maximum vertex-degree three is* **NP**-*complete.*

Proof. We show that the planar vertex-disjoint paths problem reduces to its restriction to instances $G = (V, S \cup D)$ with maximum vertex-degree three. This

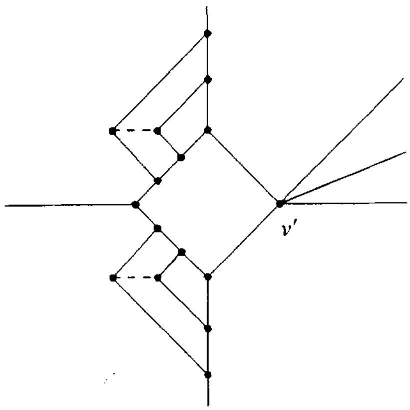

Fig. 2. $\deg(v') = \deg(v) - 1$

reduction is done by replacing vertices v of degree greater than three by gadgets as depicted in Figure 2. Notice that this gadget contains two new demand edges, depicted by dashed lines.

Observe that every path passing through the gadget has to pass the vertex v'. Thus the original graph is feasible if and only if the modified one is. ∎

Since the vertex-disjoint paths problem and the edge-disjoint paths problem coincide for graphs with maximum vertex-degree three, we obtain as a corollary

Theorem 3. *The planar edge-disjoint paths problem is* **NP**-*complete.* ∎

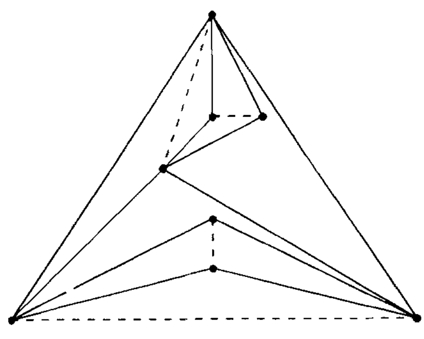

Fig. 3. A counterexample to the conjecture of A. Frank

A. Frank (as mentioned in Sebő [7]) had a conjecture on the sufficiency of a certain necessary condition for the planar edge-disjoint path problem which would immediately imply the existence of a polynomial algorithm for this problem. Assuming **P** $\neq$ **NP** this would contradict our result. However, Frank's conjecture is false as we will next show by presenting a counterexample. Let us first restate the conjecture in an equivalent formulation. For a solution $C_1,\ldots,C_{|D|}$ of an instance $G=(V,S\cup D)$ of the edge-disjoint paths problem $G-\bigcup_{d\in D}C_d$ is a T-join with T the set of vertices of odd degree in G (recall that for an even subset T of the vertex-set of a graph H a T-join is a subgraph H_T of H such that deg_{G_T} is odd exactly for vertices in T). Now an obvious necessary condition for the feasibility of a given instance G is the feasibility of the (straightforward defined) fractional relaxation of the circuit and T-join packing problem arising from the above observation. The conjecture was that this necessary condition is also sufficient. A counterexample to this conjecture is shown in Figure 3.

Theorem 4. *The edge-disjoint path problem restricted to planar instances $G=(V,S\cup D)$ where the demand-edges belong to a bounded number of faces of $G_S=(V,S)$ is polynomially solvable.*

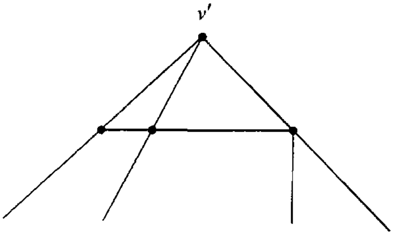

Fig. 4. $\deg(v')=\deg(v)-1$

Proof. Replacing the vertices of degree greater than four by the gadgets of Figure 4 reduces the planar edge-disjoint paths problem to its restriction to instances with maximum degree four. Observe that this replacement does not increase the number of faces of $G_S=(V,S)$ which are incident with demand-edges.

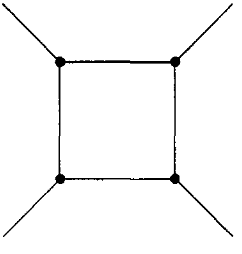

Fig. 5. Reduction of vertex-degree four

It is easy to see that if an instance of the planar edge-disjoint path problem is feasible at all then there always exists a solution without crossings. For this reason every vertex of degree four can be replaced without losing feasibility by the gadget of Figure 5. Thus the problem reduces to its restriction to instances with maximum degree three. Since for graphs with maximum degree three the edge-disjoint and the vertex-disjoint version of the problem coincide, the result follows from a theorem by A. Schrijver [6] which implies that the planar vertex-disjoint paths problem restricted to instances where the demand-edges belong to a bounded number of faces of the supply-graph (V,S) is polynomial time solvable. ∎

4. The disjoint paths problem in minor closed classes of graphs

Theorem 1 and Theorem 3 yield a complete characterization of the minor-closed classes of graphs for which the vertex-/edge-disjoint paths problem is polynomially solvable.

Any minor-closed class of graphs can be characterized by a set of excluded minors. Excluding only nonplanar minors yields a class of graphs which certainly contains all planar graphs. By Theorem 1 and Theorem 3, respectivly, both the vertex-and edge-disjoint paths problem are **NP**-complete for such classes of graphs. If at least one planar graph is excluded, we can use the following well-known theorem by N. Robertson and P. Seymour [5]:

For every planar graph H there exists a number w such that every graph G that does not contain a minor isomorphic to H has tree-width $\leq w$.

Recall that a graph G has tree-width $\leq w$ if there is a tree $T=(V,E)$ and a family $\{T_i | i \leq |V(G)|\}$ of subtrees of T with $|\{i | v \in V(T_i)\}| \leq w+1, v \in V$, such that G is isomorphic to a subgraph G' of the intersection graph G^* of the trees $T_i, i \leq |V(G)|$, i.e.
$G \simeq G' \subseteq G^* := (\{T_i | i \leq |V(G)|\}, \{\{T_i,T_j\} | V(T_i) \cap V(T_j) \neq \emptyset, i \neq j\})$. $(T, \{T_i | i \leq |V(G)|\})$ is called a tree-decomposition of G of width $\leq w$.

Graphs G of low tree-width can be cutted down by an abundance of low order separations, and these can be used to solve many problems efficiently. In particular the vertex-disjoint paths problem restricted to instances $G=(V,S \cup D)$ with tree-width bounded by some constant w can be solved in polynomial time (cf. J. Matoušek and R. Thomas [3]). In [4] we prove polynomial time solvability for the edge-disjoint paths problem in classes of graphs with bounded tree-width. In summary we get the following

Theorem 5. *(Assume* **P** $\neq$ **NP***) The vertex-disjoint paths problem restricted to instances $G=(V,S \cup D)$ of some minor-closed class $\mathcal{G}$ of graphs is polynomially solvable if and only if some planar graph H does not belong to $\mathcal{G}$.* ∎

Theorem 6. *(Assume* **P** $\neq$ **NP***) The edge-disjoint paths problem restricted to instances $G=(V,S \cup D)$ of some minor-closed class $\mathcal{G}$ of graphs is polynomially solvable if and only if some planar graph H does not belong to $\mathcal{G}$.* ∎

5. Half-integral relaxations and disjoint paths in Eulerian graphs

In [10] P. Seymour proved that for Eulerian instances $(G=(V,S\cup D),c,r)$ of the integral multicommodity flow problem (instances with $\sum_{v\in e\in E(G)}(c(e)+r(e))$ even, $v\in V$) the usual cut condition ($c(C\cap S)\geq r(C\cap D)$, where C is an edge-cut set in G) is necessary and sufficient for the feasibility of the given Eulerian instance provided that G has no subgraph contractible to K_5. In [9] he had conjectured that the existence of any solution to the multicommodity flow problem for an arbitrary instance implies the the feasibility of the instance for the half-integral multicommodity flow problem. This would imply polynomiality for the half-integral multicommodity flow problem. Seymour's conjecture was disproved by Lomonosov [2] who presented for every integer k a feasible instance of the multicommodity flow problem with $|D|=3$, such that the value $\frac{2}{k}$ is inevitably taken by some circuit in any of its solutions. Nevertheless to decide the status of complexity of the half-integral multicommodity flow problem remained an open problem. We will prove the **NP**-completeness of this problem.

Before doing so we would like to consider the vertex-disjoint paths problem once more. As in the edge-disjoint case there is a natural relaxation of the vertex-disjoint paths problem which may be formulated as a linear program again.

Let $G=(V,S\cup D)$ be a graph and A be the $0,1-$matrix with rows corresponding to vertices $v\in V$ and columns corresponding to the circuits of G with $|E(C)\cap D|=1$. An entry (v,C) is 1 if and only if $v\in V(C)$. Similarly let B be the 0,1-matrix with rows corresponding to the edges $d\in D$ and columns corresponding to the circuits from above. An entry (d,C) is 1 if and only if d belongs to $E(C)$. G is feasible for the (fractional) relaxation of the vertex-disjoint path problem if the linear program $Ax\leq 1, Bx=1, x\geq 0$ has a fractional solution.

Whereas the half-integral relaxation of the planar edge-disjoint paths problem is polynomially solvable (as mentioned above) we can prove for the vertex-disjoint case the following

Theorem 7. *The half-integral relaxation of the planar vertex-disjoint paths problem is* **NP***-complete.*

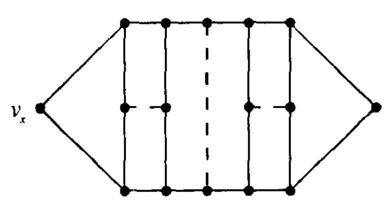

Fig. 6. G_v

Proof. We will reduce the planar vertex-disjoint paths problem which is **NP**-complete by Theorem 1 to its half-integral relaxation. Let an instance $G=(V,S\cup D)$ be given. For every demand-edge $d\in D$ we add a parallel edge. Then we subdevide these new edges once. Let x_d denote the vertex subdividing the new edge in parallel to d. For every $x \in V \cup \{x_d | d \in D\}$ not incident with an edge $d \in D$ take a copy of the gadget G_v depicted in Figure 6 (dashed lines refer to demand edges) and identify the vertices x and v_x.

Let $G' = (V', S' \cup D')$ denote the resulting graph where D' contains exactly the demand edges of G and the demand edges of the gadgtes. In any solution the vertices v_x of the gadgets belong to exactly one circuit that contains a gadget-demand-edge. This circuit is taken $\frac{1}{2}$ times. Using this observation it is easy to see that original graph G is a feasible instance of the planar vertex-disjoint paths problem if and only if G' is a feasible instance of its half-integral relaxation. ∎

Theorem 8. *The half-integral multicommodity flow problem is* **NP**-*complete.*

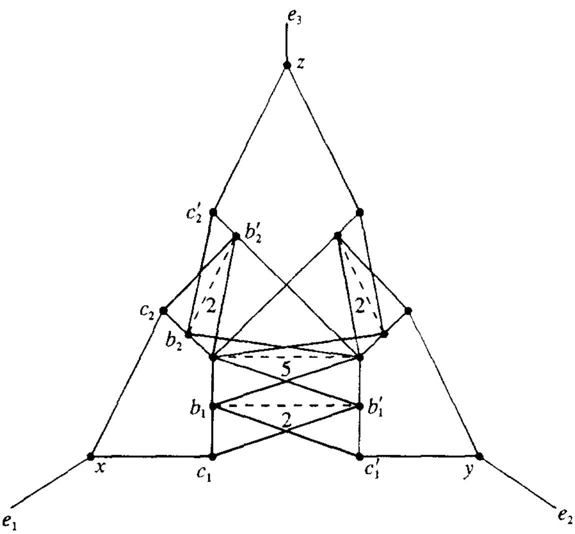

Fig. 7. Vertex-gadget

Proof. We give a reduction of the vertex-disjoint paths problem in cubic graphs to the half-integral multicommodity flow problem. Let $G = (V, S \cup D)$ be a cubic graph. From G we build the graph $G' = (V', S' \cup D')$ by replacing the vertices of G by gadgets as depicted in Figure 7 in such a manner that the edges e_1, e_2, e_3 correspond to the edges of G and such that edges corresponding to demand-edges of D together with the dotted edges in the gadgets form the set D' of demand edges of G'.

We claim that G is a feasible instance of the vertex-disjoint paths problem if and only if G' is a feasible instance of the half-integral multicommodity flow problem.

Add one additional demand edge between two of the vertices x, y, z to the gadget. Observe that the resulting graph is a feasible instance of the half-integral multicommodity flow problem. By this observation it is obvious that one can easily construct from a solution of G (G is an instance of the vertex-disjoint paths problem) a solution of G' (G' is an instance of the half-integral multicommodity flow problem).

In order to prove that feasibilty of G' implies feasibility of G we need some more observations about our gadget.

Let S_{gad} denote the set of supply-edges of one fixed gadget gad in G' not incident to one of the vertices x,y,z. Let D_{gad} denote the set of demand-edges of gad.

Every circuit C in G' with $|C \cap D'| = 1$ that contains an edge of gad contains at least two edges of S_{gad}. Since gad has 11 demand-edges and S_{gad} has 24 elements at most either one circuit containing a demand edge of $D' - D_{gad}$ that passes gad with value one or two such circuits with value 1/2 can be used in a half-integral solution of G'. Thus it is easy to see that for our goal it suffices to show that a solution of the half-integral multicommodity flow problem for G' cannot use two circuits C_1 and C_2 containing demand-edges from $D' - D_{gad}$ both with value 1/2 entering gad via e_1, say, such that C_1 leaves gad via e_2 whereas C_2 leaves via e_3.

Assume for a contradiction such a solution SOL exists.

By the above counting argument both C_1 and C_2 can contain only two edges of S_{gad}. Since every circuit C with $|C \cap D_{gad}| = 1$ that contains a supply-edge not in S_{gad} contains at least four edges of S_{gad}, every circuit C containing a demand edge of D_{gad} used in SOL lies completely in $S_{gad} \cup D_{gad}$. By the symmetry of $S_{gad} \cup D_{gad}$ we can assume that C_1 contains the edges $c_1 b_1$ and $b_1 c_1'$ whereas C_2 contains the edges $c_2 b_2$ and $b_2 c_2'$. Now it follows that SOL can use only one circuit with value 1/2 that contains the edge $c_1 b_1'$. Our counting argument yields the desired contradiction that finishes the proof of the theorem. ∎

Corollary 1. *The edge-disjoint paths problem in Eulerian graphs is* **NP***-complete.* ∎

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